Cambridge International AS & A Level

MATHEMATICS **9709/42**

Paper 4 Mechanics **May/June 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1: Marks must be awarded in line with: • the specific content of the mark scheme or the generic level descriptors for the question • the specific skills defined in the mark scheme or in the generic level descriptors for the question • the standard of response required by a candidate as exemplified by the standardisation scripts.
GENERIC MARKING PRINCIPLE 2: Marks awarded are always **whole marks** (not half marks, or other fractions).
GENERIC MARKING PRINCIPLE 3: Marks must be awarded **positively**: • marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate • marks are awarded when candidates clearly demonstrate what they know and can do • marks are not deducted for errors • marks are not deducted for omissions • answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.
GENERIC MARKING PRINCIPLE 4: Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.
GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1	Initial KE $= \frac{1}{2} \times 0.6 \times 4^2$ $[= 4.8]$ Final KE $= \frac{1}{2} \times 0.6 \times v^2$ PE loss $= 0.6 \times g \times 15 \sin 10$ $[= 15.628]$	**B1**	Any one of the three expressions correct
	$0.6 \times g \times 15 \sin 10 + \frac{1}{2} \times 0.6 \times 4^2 = \frac{1}{2} \times 0.6 \times v^2$	**M1**	Apply energy equation, 3 terms, dimensions correct
	$v = 8.25$ ms^{-1}	**A1**	
		3	

Question	Answer	Marks	Guidance
2	Resolve either horizontally or vertically with correct number of terms.	**M1**	Allow θ and α as in the question for this mark
	$[X =] 30 - 34 \times \frac{8}{17} - 26 \times \frac{5}{13} [= 4]$	**A1**	Allow $\pm X$ as they may resolve forces left or right Allow $[X =] 30 - 34 \sin 28 - 26 \sin 23$ angle 2s.f. or better
	$[Y =] 34 \times \frac{15}{17} - 26 \times \frac{12}{13} [= 6]$	**A1**	Allow $\pm Y$ as they may resolve forces up or down Allow $[Y =]$ $34 \cos 28 - 26 \cos 23$ angle 2s.f. or better
	$[R =] \sqrt{X^2 + Y^2}$	**M1**	Attempt to solve for the magnitude of the force
	$[\beta =] \tan^{-1}\left(\frac{Y}{X}\right)$ or $[\beta =] \tan^{-1}\left(\frac{X}{Y}\right)$	**M1**	Attempt to solve for the direction of the resultant force

Question	Answer	Marks	Guidance
2 cont'd	$R=\sqrt{52}=2\sqrt{13}=7.21$ N **and** $\beta=56.3$ above 30N force or anticlockwise from 30N force	**A1**	Both correct with correct explanation of the direction. Must be a correct and clear explanation.
		6	

Question	Answer	Marks	Guidance
3	Resolving along or perpendicular to the rod	**M1**	3 terms in either direction
	$8\sin 10 + R = 0.3g$	**A1**	
	$8\cos 10 - F = 0.3a$	**A1**	
	$F = 0.8R$ $[R = 1.61081\ldots, F = 1.28865\ldots]$	**M1**	Using $F = \mu R$, where R is 2 terms involving weight and a component of 8 N.
	$[a = 21.966\ldots]$ $0.6 = \frac{1}{2} \times 21.966 \times t^2$	**M1**	Complete method leading to an equation in t such as $s = ut + \frac{1}{2}at^2$ with $s = 0.6$, $u = 0$ and using *their* value of a found from a Newton's second law with 3 terms, namely, component of 8 N, any friction and $0.3a$.
	$t = 0.234$ seconds	**A1**	Allow use of $a = 22$ for M1 and A1
	Alternative method for Question 3		
	Resolving perpendicular to the rod	**M1**	
	$8\sin 10 + R = 0.3g$	**A1**	
	$F = 0.8R$ $[R = 1.61081\ldots, F = 1.28865\ldots]$	**M1**	Using $F = \mu R$, where R must involve $0.3g$ and a component of 8 N.

Question	Answer	Marks	Guidance
3	$8\cos 10 \times 0.6 = F \times 0.6 + \frac{1}{2} \times 0.3v^2 \quad [v = 5.134]$	B1	Work energy equation to find v after 0.6 metres.
	$0.6 = \frac{1}{2}(0 + 5.134) \times t$	M1	Using $s = \frac{1}{2}(u+v)t$ to find t.
	$t = 0.234$ seconds	A1	
		6	

Question	Answer	Marks	Guidance
4	For resolving either parallel to or perpendicular to the plane	M1	Three relevant terms in either equation.
	$P\cos 8 = F + 12g\sin 25$	A1	
	$12g\cos 25 = R + P\sin 8$	A1	
	$F = 0.3R$	M1	Use $F = 0.3R$, where R must involve components of both $12g$ and P.
	$P\cos 8 = 0.3(12g\cos 25 - P\sin 8) + 12g\sin 25$	M1	For attempting to solve for P, using equations with the correct number of relevant terms in both.
	$P = 80.8$	A1	From $P = 80.755\ldots$ Allow $P \leqslant 80.8$ If more than one case is considered for direction of friction then a choice must be made for final answer.
	Alternative mark scheme for Question 4		
	For resolving forces either vertically or horizontally	M1	Correct number of terms in either equation.
	$R\cos 25 + P\sin 33 = 12g + F\sin 25$	A1	

Question	Answer	Marks	Guidance
4	$P\cos 33 = F\cos 25 + R\sin 25$	**A1**	
	$F = 0.3R$	**M1**	Use $F = 0.3R$
	Solve a pair of simultaneous equations in P and R May see $R = 97.5$	**M1**	For attempting to solve for P, using equations with the correct number of relevant terms.
	$P = 80.8$	**A1**	From $P = 80.755\ldots$ Allow $P \leqslant 80.8$ If more than one case is considered for direction of friction then a choice must be made for final answer.
		6	

Question	Answer	Marks	Guidance
5(a)(i)	$P = (440 + 280) \times 30$	**M1**	Using $P = Fv$ with F as total resistance
	$P = 720 \times 30 = 21.6\ \text{kW}$	**A1**	Answer must be in kW
		2	

Question	Answer	Marks	Guidance
5(a)(ii)	$P = 21600 - 8000$ W $\text{DF} = \frac{21600-8000}{30}$ $\left[= \frac{13600}{30} = 453.333..\right]$	B1 FT	Follow through on *their* power from **5(a)(i)** Allow Driving Force $(\text{DF}) = \frac{8000}{30} = 266.7$ as the force due to solely to the change in power provided correct equation(s) used.
	Car: $\text{DF} - 440 - T = 1250a$ Caravan: $T - 280 = 800a$ System: $\text{DF} - (440 + 280) = 2050a$	M1	Apply Newton's 2nd law to either the car or to the caravan or to the system. Must be correct number of relevant terms. If $\text{DF} = \frac{8000}{30}$ is used then the equations must be either $-\text{DF} = 2050a$ or $T - 280 = 800a$
	Solve for either a or T	M1	Using equation(s) with no missing/extra terms, $\text{DF} \neq 720$. Solving for a either from the system equation or from the car AND caravan equation. OR solving for T from the car AND caravan equation.
	$a = -0.13$ ms^{-2} and $T = 176$ N	A1	
		4	

Question	Answer	Marks	Guidance
5(b)(i)	System: $\text{DF} = 720 + 2050g \times 0.06$ $[= 1950]$ Car: $\text{DF} - 440 - T - 1250g \times 0.06 = 0$ Caravan: $T - 280 - 800g \times 0.06 = 0$	M1	Apply Newton's 2nd law with $a = 0$, either to the system OR by eliminating T between the equations for the car and the caravan, no extra or missing relevant terms, dimensionally correct, to find DF
	$1950v = 28000$	B1	$P = \text{DF} \times v$. $\frac{28000}{v}$ SOI.
	$v = 14.4$ ms^{-1}	A1	
		3	

Question	Answer	Marks	Guidance
5(b)(ii)	$\text{PE} = 800g \times d \times 0.06 = 800g \times 14.4 \times 60 \times 0.06$	**M1**	Using PE = *mgh* with *h* being height gained in 60 s, using *their* *v*
	$\text{PE} = 414\,000$ (J) or $\text{PE} = 414$ kJ	**A1**	Using *v* = 560/39 = 14.359
	Alternative method for Question 5(b)(ii)		
	$28\,000 \times 60 = \text{PE of Caravan} + 1250g \times d \times 0.06 + 720 \times d$ **and** $d = 60 \times 14.359 = 861.54$	**M1**	For use of $\text{WD} = P \times t$ to find an expression for PE of caravan and the distance travelled up the incline in 1 minute.
	$[\text{PE} = 28\,000 \times 60 - 1250g \times 861.54 \times 0.06 - 720 \times 861.54]$ $\text{PE} = 414\,000$ (J) or $\text{PE} = 414$ kJ	**A1**	
		2	

Question	Answer	Marks	Guidance
6	$s_A = \pm(30t - 5t^2)$ or $s_B = \pm 5t^2$	**B1**	Use of constant acceleration equations to find expressions for displacements of *A* or *B*.
	$s_A + s_B = 15$ leading to $15 = 30t$ leading to $t = 0.5$	**B1**	Use $s_A + s_B = 15$ to find time at which particles collide.
	$t = 0.5$ leading to $v_A = \pm 25$ and $v_B = \pm 5$	**B1**	Find speed of particles at *t* = 0.5 before collision.
	$t = 0.5$ leading to $h_A = \pm\left(30 \times 0.5 - \frac{1}{2}g \times 0.5^2\right) = \pm 13.75$	**B1**	Find position of *A* or *B* at which collision occurs at *t* = 0.5 Alternatively allow $h_B = \pm 1.25$ as displacement of *B*
	$25 \times (2m) - 5(m) = (3m)v \rightarrow v_1 = 15$ $25(m) - 5 \times (2m) = (3m)v \rightarrow v_2 = 5$	**M1**	Use of conservation of momentum, either case, using *their* v_A and $v_B \neq 0$ or 30, with 3 terms.
		A1	Both values of v correct

Question	Answer	Marks	Guidance
6	Particle C_1 $-13.75 = 15t - 5t^2$ Particle C_2 $-13.75 = 5t - 5t^2$	M1	Use of $s = ut + \frac{1}{2}at^2$ OE to find t, using either *their* numerical v_1 or numerical v_2 from a relevant conservation of momentum equation.
	$t_{C_1}, t_{C_2} = 3.74, 2.23$ leading to $T = 1 + \sqrt{5} - \sqrt{3} = 1.50$	A1	Find $T = t_{C_1} - t_{C_2}$ from $t_{C_1} = 3.736$ and $t_{C_2} = 2.232$
		8	Subscripts 1 and 2 refer to the two cases.
	Alternative method for the final two marks		
	$0 = 15 - gt_1$, $0 = 5 - gt_2 \rightarrow t_1 = 1.5$, $t_2 = 0.5$ Total heights $h_1 = 13.75 + 11.25 = 25$ Or $h_2 = 13.75 + 1.25 = 15$ $25 = 5T_1^2$ and $15 = 5T_2^2 \rightarrow T_1 = \sqrt{5}$, $T_2 = \sqrt{3}$	M1	Use of $v = u - gt$ to find time to highest point for either case and use of $v^2 = u^2 - 2gs$ to find total height reached for either case, using either *their* numerical v_1 or numerical v_2 from a relevant conservation of momentum equation. Use $s = 0 + \frac{1}{2}gT^2$ to find time to reach ground (either case).
	$T = 1.5 + \sqrt{5} - (0.5 + \sqrt{3}) = 1 + \sqrt{5} - \sqrt{3} = 1.50$	A1	Find difference in total times $T = (t_1 + T_1) - (t_2 + T_2)$

Question	Answer	Marks	Guidance
7(a)	$v = 6t + 2t^2 [+c]$ or $v = 14t[+c]$	**M1**	Attempt to integrate *a* in Stage 1 or Stage 2 or in Stage 2 for use of $v = u + at$
	$v = 6t + 2t^2$ and $v = 14t - 8$ or $v(t = 2) = 20$ $v(t = 4) = 20 + 14 \times 2 = 48$	**A1**	Velocity in Stage 1 and Stage 2 correct including correct constant Find v at $t = 2$ and use $v = u + 14t$ to find v at $t = 4$
	$v = 16t - t^2 [+c]$	***M1**	Attempt to integrate a in Stage 3.
	$55 = 16t - t^2$	**DM1**	Attempt to solve a relevant 3-term quadratic equation which comes from their 2 term v from Stage 3 equated to 55 and finding two values of t
	$t = 5$ and $t = 11$ only	**A1**	Allow only if $c = 0$ has been shown correctly.
	Alternative method for Question 7(a)		
	State or imply that only possible range is $4 \leqslant t \leqslant 16$	**B1**	Allow this method if candidates only consider Stage 3
	$v = 16t - t^2 + c$	**M1**	For attempt at integration.
	$c = 0$ shown	**A1**	Using $v = 0$ at $t = 16$
	Solve $55 = 16t - t^2$	**M1**	Must find 2 values of t and must be from equating *their* 2 term v to 55
	$t = 5$ and $t = 11$ only	**A1**	Allow only if $c = 0$ has been shown correctly.
		5	

Question	Answer	Marks	Guidance
7(b)	Positive quadratic for $0 \leqslant t < 2$ through (0,0) joining to the bottom of the given line or Negative quadratic for $4 \leqslant t \leqslant 16$ going through the point (16,0) and joining the top of the given line	**B1**	
	All correct with correct gradients (approx)	**B1**	Negative quadratic must have a maximum. There must be no point of inflexion particularly near $t = 16$. Ignore any curve drawn outside $0 \leqslant t \leqslant 16$.
		2	
7(c)	$s = \int \left(16t - t^2\right) \mathrm{d}t \quad \left[= 8t^2 - \frac{1}{3}t^3 \left(+c\right)\right]$	**M1**	Attempt to integrate *their v*.
	$s = \left[8t^2 - \frac{1}{3}t^3\right]_8^{16}$ $s = \left[2048 - 1365\frac{1}{3}\right] - \left[512 - 170\frac{2}{3}\right]$	**A1**	Correct integral and the correct limits used correctly to find an unsimplified expression for the distance from $t = 8$ to $t = 16$ only.
	$s = 341\frac{1}{3}$	**B1**	Allow $s = 341$ to 3s.f. If no integration seen (calculator used) allow B1 (max 1 out of 3 marks)
		3	